Chapter - Statistics

Topic-1: Arithmetic Mean, Geometric Mean, Harmonic Mean, Median & Mode

1 MCQs with One Correct Answer

1. If $x_1, x_2, \ldots\ldots\ldots\ldots, x_n$ are any real numbers and n is any postive integer, then [1982 - 2 Marks]

(a) $n\sum_{i=1}^{n} x_i^2 < \left(\sum_{i=1}^{n} x_i\right)^2$ (b) $\sum_{i=1}^{n} x_i^2 \geq \left(\sum_{i=1}^{n} x_i\right)^2$

(c) $\sum_{i=1}^{n} x_i^2 \geq n\left(\sum_{i=1}^{n} x_i\right)^2$ (d) none of these

4 Fill in the Blanks

2. A variable takes value x with frequency ${}^{n+x-1}C_x$, $x = 0, 1, 2, \ldots n$. The mode of the variable is................. [1982 - 2 Marks]

6 MCQs with One or More than One Correct Answer

3. In a college of 300 students every student reads 5 newspapers and every newspaper is read by 60 students. The number of newpapers is [1998 - 2 Marks]

(a) at least 30 (b) at most 20

(c) exactly 25 (d) none of these

Topic-2: Quartile, Measures of Dispersion, Quartile Deviation, Mean Deviation, Variance & Standard Deviation, Coefficient of Variation

1 MCQs with One Correct Answer

1. Consider any set of 201 observations $x_1, x_2, \ldots x_{200}, x_{201}$. It is given that $x_1 < x_2 < \ldots < x_{200} < x_{201}$. Then the mean deviation of this set of observations about a point k is minimum when k equals [1981 - 2 Marks]

(a) $(x_1 + x_2 + \ldots + x_{200} + x_{201})/201$

(b) x_1

(c) x_{101}

(d) x_{201}

2. The standard deviation of 17 numbers is zero. Then [1980]

(a) the numbers are in geometric progression with common ratio not equal to one.

(b) eight numbers are positive, eight are negative and one is zero.

(c) either (a) or (b)

(d) none of these

3. Select the correct alternative in each of the following. Indicate your choice by the appropriate letter only. Let S be the standard deviation of n observations. Each of the n observations is multiplied by a constant c. Then the standard deviation of the resulting number is [1980]

(a) s (b) $c\,s$

(c) $s\sqrt{c}$ (d) none of these

7 Match the Following

4. Consider the given data with frequency distribution [Adv. 2023]

x_i	3	8	11	10	5	4
f_i	5	2	3	2	4	4

Match each entry in List-I to the correct entries in List-II.

List-I	List-II
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5

(R) The mean deviation about the mean of the above data is (3) 6

(S) The mean deviation about the median of the above data is (4) 2.7

(5) 2.4

The correct option is:

(a) (P) → (3), (Q) → (2), (R) → (4), (S) → (5)
(b) (P) → (3), (Q) → (2), (R) → (1), (S) → (5)
(c) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(d) (P) → (3), (Q) → (3), (R) → (5), (S) → (5)

10 Subjective Problems

5. The marks obtained by 40 students are grouped in a frequency table in class intervals of 10 marks each. The mean and the variance obtained from this distribution are found to be 40 and 49 respectively. It was later discovered that two observations belonging to the class interval (21–30) were included in the class interval (31–40) by mistake. Find the mean and the variance after correcting the error. **[1982 - 3 Marks]**

6. The mean square deviations of a set of observations $x_1, x_2, \ldots, x_n$ about a points c is defined to be $\frac{1}{n}\sum_{i=1}^{n}(x_1 - c)^2$. The mean suqare deviations about –1 and +1 of a set of observatons are 7 and 3 respectively. Find the standard deviation of this set of observations. **[1981 - 2 Marks]**

7. In calculating the mean and variance of 10 readings, a student wrongly used the figure 52 for the correct figure of 25. He obtained the mean and variance as 45.0 and 16.0 respectively. Determine the correct mean and variance. **[1979]**

Answer Key

Topic-1 : Arithmetic Mean, Geometric Mean, Harmonic Mean, Median & Mode

1. (d) 2. (n) 3. (c)

Topic-2 : Quartile, Measures of Dispersion, Quartile Deviation, Mean Deviation, Variance & Standard Deviation, Coefficient of Variation

1. (c) 2. (d) 3. (b) 4. (a)

Hints & Solutions

Topic-1: Arithmetic Mean, Geometric Mean, Harmonic Mean, Median & Mode

1. (d) We know that mth power mean inequality that

$$\frac{x_1^m + x_2^m + \ldots + x_n^m}{n} \ge \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)^m$$

$$\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n} \ge \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right)^2$$

$$\Rightarrow \frac{n^2}{n}\left(\sum_{i=1}^{n} x_i^2\right) \ge \left(\sum_{i=1}^{n} x_i\right)^2 \Rightarrow n\left(\sum_{i=1}^{n} x_i^2\right) \ge \left(\sum_{i=1}^{n} x_i\right)^2$$

2. Given that frequency for variable x is ${}^{n+x-1}C_x$ where $x = 0, 1, 2, \ldots\ldots n$.

Mode is the variable for which frequency is maximum.

We know that, if n is even then ${}^nC_r$ is max. for $r = n/2$,

if n is odd then $r = \frac{n+1}{2}$

If $n + x - 1$ is even then for maximum value of ${}^{n+x-1}C_x$,

$$x = \frac{n+x-1}{2} \Rightarrow x = n-1, \therefore \text{ frequency} = {}^{2n-2}C_{n-1}$$

If $n + x - 1$ is odd then for maximum value of ${}^{n+x-1}C_x$

$$x = \frac{n+x-1+1}{2} \Rightarrow x = n, \therefore \text{ frequency} = {}^{2n-1}C_n$$

But we have ${}^{2n-1}C_n = \frac{2n-1}{n}\,{}^{2n-2}C_{n-1}$

i.e., ${}^{2n-1}C_n > {}^{2n-2}C_{n-1}$. So, ${}^{2n-1}C_n$ is maximum frequency.

$\therefore$ Mode should be n.

3. (c) Let the number of newspapers which are read be n. Then $60\,n = (300)\,(5) \Rightarrow n = 25$

4. (a) Given

x_i	3	4	5	8	10	11
f_i	5	4	4	2	2	3

x_i	f_i	$x_i f_i$	C.F.	$\lvert x_i - \text{Mean}\rvert$	$f_i \lvert x_i - \text{Mean}\rvert$	$\lvert x_i - \text{Median}\rvert$	$f_i \lvert x_i - \text{Median}\rvert$
3	5	15	5	3	15	2	10
4	4	16	9	2	8	1	4
5	4	20	13	1	4	0	0
8	2	16	15	2	4	3	6
10	2	20	17	4	8	5	10
11	3	33	20	5	15	6	18
	$\Sigma f_i = 20$	$\Sigma x_i f_i = 120$			$\Sigma f_i \lvert x_i - \text{Mean}\rvert = 54$		$\Sigma f_i \lvert x_i - \text{Median}\rvert = 48$

(P) Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{120}{20} = 6$

(Q) Median $= \left(\frac{N}{2}\right)^{th}$ obs. $\left(\frac{20}{2}\right)^{th}$ obs. $= 10^{th}$ obs. $= 5$

(R) Mean deviation about mean

$$= \frac{\Sigma f_i \lvert x_i - \text{Mean}\rvert}{\Sigma f_i} = \frac{54}{20} = 2.70$$

(S) Mean deviation about median

$$= \frac{\Sigma f_i \lvert x_i - \text{Median}\rvert}{\Sigma f_i} = \frac{48}{20} = 2.40$$

Topic-2: Quartile, Measures of Dispersion, Quartile Deviation, Mean Deviation, Variance & Standard Deviation, Coefficient of Variation

1. (c) Given that $x_1 < x_2 < x_3 < \ldots < x_{201}$

$\therefore$ Median of the given observation $= \frac{201+1}{2}$ th obs.

$= 101^{th}$ obs. $= x_{101}$

We know that, deviations will be minimum if taken from the median

$\therefore$ Mean deviation will be minimum if $k = x_{101}$.

2. (d) If s. d. = 0, statements (a) and (b) can not be true.

3. (b) We know that if each of n observations is multiplied by a constant c, then the standard deviation also gets multiplied by c.

5. $n = 40,\ \bar{x} = 40, \text{Var.} = 49$

$$\frac{\sum f_i x_i}{40} = \bar{x} = 40 \Rightarrow \sum f_i x_i = 1600 \quad \text{....(i)}$$

$$\text{Variance} = \frac{\Sigma x_i^2 f_i}{40} - (40)^2;\quad 49 + 1600 = \frac{\Sigma x_i^2 f_i}{40}$$

$$\Rightarrow \frac{1}{40}\sum f_i x_i^2 = 1649 \quad \text{...(ii)}$$

Let 21 – 30 and 31 – 40 denote the k^{th} and $(k+1)^{th}$ class intervals respectively with frequency f_k and f_{k+1} since, 2 observations are shifted from 31 – 40 to 21 – 30 therefore frequency of k^{th} intervals becomes $f_k + 2$ and frequency of $(k+1)^{th}$ interval becomes $f_{k+1} - 2$.

Then, we get

$$\bar{x}_{new} = \frac{1}{40}\sum_{i=1}^{40} f_i x_i + \frac{2}{40}(x_k - x_{k+1})$$

$$= \frac{1}{40}\sum_{i=1}^{40} f_i x_i + \frac{1}{20}(-10) = 40 - 0.5 = 39.5$$

$$(Var)_{new} = \left[\frac{\Sigma x_i^2 f_i}{40} + \frac{2}{40}\left(x_k^2 - x_{k+1}^2\right)\right] - (39.5)^2$$

$$= 1649 + \frac{2(25.5)^2 - (35.5)^2)}{40} - (39.5)^2$$

$$= 1649 + 2\frac{(-10)(61)}{40} - (39.5)^2$$

$$= 1649 - 30.50 - 1560.25 = 58.5$$

6. Given that mean square deviation for the observations $x_1, x_2, \ldots, x_n$, about a point c is $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$.

Also given that mean square deviations about – 1 and + 1 are 7 and 3

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}(x_i + 1)^2 = 7 \Rightarrow \sum_{i=1}^{n}(x_i^2 + 2x_i + 1) = 7n$$

$$\Rightarrow \sum x_i^2 + 2\sum x_i + n = 7n$$

$$\Rightarrow \sum x_i^2 + 2\sum x_i = 6n \quad \text{...(i)}$$

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - 1)^2 = 3;\ \sum_{i=1}^{n}(x_i^2 - 2x_i + 1) = 3n$$

$$\Rightarrow \sum x_i^2 - 2\sum x_i + n = 3n$$

$$\text{and } \sum x_i^2 - 2\sum x_i = 2n \quad \text{...(ii)}$$

Subtracting (ii) from (i), we get

$$4\sum x_i = 4n \Rightarrow \frac{\sum x_i}{n} = 1 \Rightarrow \bar{x} = 1$$

Now standard deviation for same set of observations

$$= \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - 1)^2} = \sqrt{3}$$

7. $\text{Mean} = \frac{\Sigma x_i}{N} \Rightarrow 45 = \frac{\Sigma x_i}{10} \quad \therefore \Sigma x_i = 450$

Correct $\Sigma x_i = 450 - 52 + 25 = 423$

$\therefore$ Correct mean $= \frac{423}{10} = 42.3$;

$$\text{Variance} = \frac{\Sigma x_i^2}{N} - \left(\frac{\Sigma x_i}{N}\right)^2;\ 16 = \frac{\Sigma x_i^2}{10} - (45)^2$$

$$\Rightarrow \Sigma x_i^2 = (16 + 2025) \times 10 = 20410$$

$\therefore$ Correct $\sum x_i^2 = 20410 - (52)^2 + (25)^2$

$= 20410 - 2704 + 625 = 18331$

$\therefore$ Correct variance $= \frac{18331}{10} - (42.3)^2$

$= 1833.1 - 1789.29 = 43.81$